THE PREDATOR-PREY RELATIONSHIP AND ITS
APPEARANCE IN STOCK MARKET TREND FLUCTUATIONS

Paul A. LaViolette

In recent years, Wall Street has been witnessing a renaissance as market analysis has begun to leave the traditional rule-of-thumb era and begins to emerge as a science. Known by the name of "efficient market theory," this new school of thought has been developing over the past two decades at the University of Chicago and is now gradually catching on with the more liberal sector of the financial community. Applied to stock market behavior, this theory states that any changes in individual stock prices occur in essentially a random fashion or, in other words, that stock prices appear to follow a random walk (Lorie 1973). Hence, any technical analysis of individual stock price movements undertaken for speculative purposes is essentially futile. The "semi-strong" form of the theory asserts that current prices of stocks fully reflect public knowledge of the underlying companies and that efforts to obtain and analyze such knowledge do not aid one in reaping superior returns.

Efficient market theory has rather conclusively explained micro-market behavior, such as the movements of individual securities or short term movements of the whole market, but it has encountered difficulty in explaining macroscopic long term movements. In the long run, the market apparently exhibits a systematic trend-cycle behavior of a non-random nature; and as Julius Shiskin (1968, pp. 670-71) points out,

> (1) while irregular fluctuations dominate the month-to-month movements in stock prices (and most other economic indicators) systematic movements dominate when longer span comparisons are made; (2) the average duration of a run in stock prices is significantly higher than that of a random series, even after the strong upward trend of recent years is eliminated; (3) diffusion indexes of stock prices computed over short spans have the irregular behavior characteristic of random series, but show systematic movements with clear cyclical amplitudes and consistent leads over aggregate stock price fluctuations when the span of comparison is extended; and (4) most important of all, stock prices consistently conform to and lead broad expansions and contractions in aggregate economic activity (the composite of such measures as total employment, income, production and trade); this feature of stock price fluctuations distinguishes it from cumulated random series (i.e., series with random first differences) which also have systematic movements in some other ways similar to those of stock price fluctuations. These systematic movements in stock prices are difficult to predict because, unlike mathematical curves, they vary in amplitude, pattern, and duration, and they are sometimes obscured by irregular fluctuations.

Shiskin (1968, p. 675) makes the following observations about the average duration of a run in stock market prices:

> For a random series, short runs occur much more frequently than long runs, and the expected average duration of run is

only 1.5 (months, quarters, or whatever the time unit in which the series is expressed). For random series with 120 observations (i.e., 10 years in monthly data) the average duration of run falls within the range 1.36 and 1.75 about 95 percent of the time.

> The average duration of run for stock prices is 2.37, well above the limits for a random series. Since stock prices had a pronounced upward trend from 1948 to 1966, the average duration of a run was also computed for this series after the trend was eliminated. It proved to be 2.30, also well above the limits for a random series.

Figures 1 and 2 show a comparison of diffusion indexes of stock prices to diffusion indexes of 24 artificial series with random first differences. The former plots show significant systematic behavior especially for spans greater than 3 months.

Shiskin has examined the relation of the stock market to other types of economic series, such as business cycles, using probability tests. He has found that during the period from 1873 to date stock prices have led the National Bureau of Economic Research reference turning
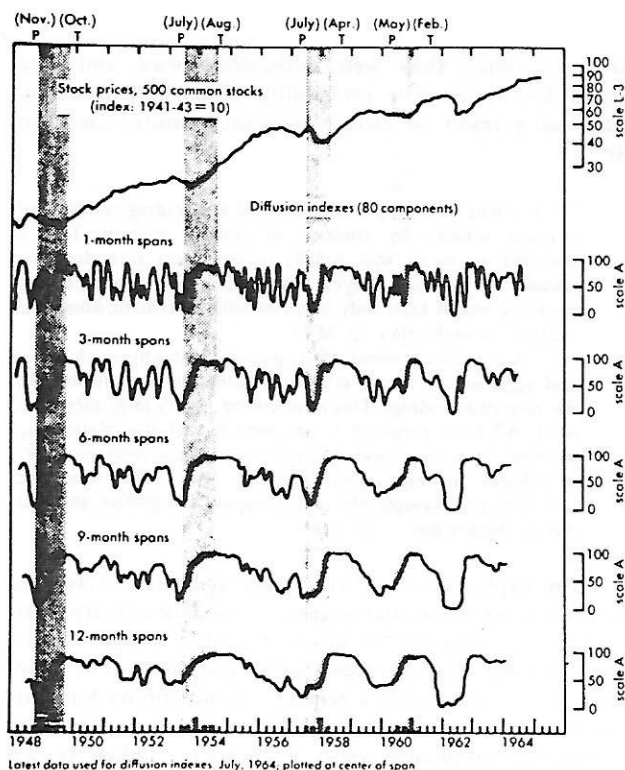

Fig. 1. Stock prices and diffusion indexes of stock prices over 1, 3, 6, 9, and 12 month spans. (From J. Shiskin in J. Lorie and R. Brealey, *Modern Developments in Investment Management*, Praeger, 1968, p. 676.)
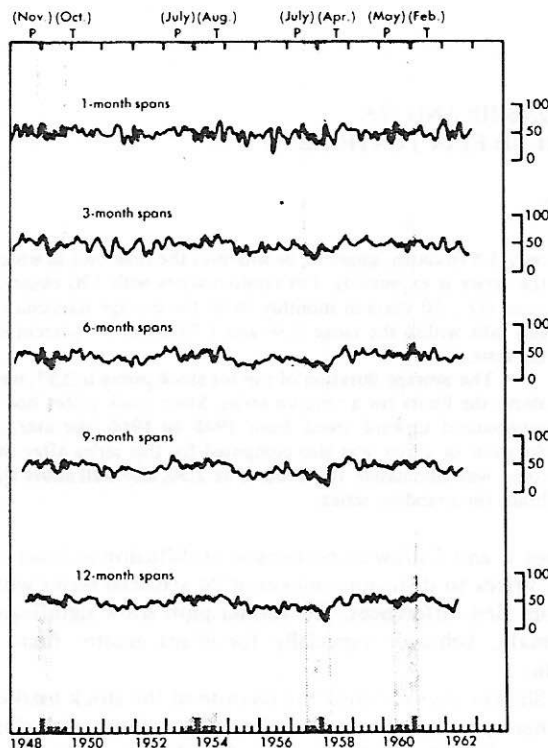
Fig. 2. Diffusion indexes of 24 artificial series with random first differences computed over 1, 3, 5, 9, and 12 month spans. (From J. Shiskin, ibid., p. 677.)

dates 33 times; they were coincident twice, and they lagged five times. The probability of having this many leads, as a result of chance, is quite remote—less than 10^{-4}.

> Stock prices have long been identified as a leading indicator of business activity by students of cyclical analysis. This is intended to mean that usually stock prices turn down in advance of a business cycle trough. Such an orderly timing sequence would exist only in series with systematic economic relations to each other. (p. 687)
>
> A convincing economic explanation of why stock prices lead aggregate economic activity is needed to lend credence to the empirical findings. This is, of course, a very large subject in itself. All I am prepared to say here is that the relationship between stock prices and aggregate economic activity probably is indirect through causal relations with other economic activities, for example, the money supply and profits, and that it is a complex one. . . (p. 686)

An explanation of why these systematic relations between stock prices and aggregate economic activity exist has not been provided by efficient market theory. In this paper, we will attempt to shed some light on some of these questions by employing a general systems approach in our examination of macroscopic stock market behavior. This illuminating approach views the stock market as an economic species whose behavior conforms to much the same laws that govern biologic species in an ecosystem. It will be found that the predator-prey fluctuations observed in the populations of biologic species are also observed to

occur in the pricing of economic species such as the stock market. By employing a biologic model whereby the stock market and other economic series are viewed as interacting species, phenomena, such as those pointed out by Shiskin above, become more easily understood. The systems approach potentially offers a new horizon of understanding in the related fields of economics and finance.

THE BIOLOGIC ANALOGY OF THE STOCK MARKET

The similarity of economic processes to biological processes has been long recognized by many notable economists and has been subject to frequent analysis from a general systems viewpoint (see Boulding 1956, Gerard 1956, Goldsmith 1971, and Spencer 1906). Reassured of the intuitive validity of this approach, let us begin by drawing an analogy between the basic competitive processes which take place both in the ecosystem and in the economy.

In the ecosystem, biologic species compete for the available organic matter. Each species attempts to organize this matter by augmenting its population. In the process, species interact with each other, indirectly by competing for a common food supply or directly through predator-prey interactions. Thus each species is both eating and being eaten. All species are struggling to organize organic matter and all are woven into a competitive fabric called the ecosystem.

By analogy, in the economy, all economic species are competing to structure money, rather than matter, into their systems. Whereas a biologic species attempts to increase its *numbers* relative to other species, an economic species attempts to increase its *value* relative to other species. Again, predator-prey behavior should be observed. For example, let us view the stock market as an economic species coexisting and interacting with other economic species, such as the bond market, commodities market, banking system, foreign markets, personal hoarding, etc. When investors shift their holdings from say, the bond market to the stock market, the stock market takes on the role of the predator species, and the bond market is here the prey. The reverse is true if investors begin investing more in the bond market. From a simplified view, when a species structures money into itself and augments in value, it has removed money from competing economic species which have consequently declined in value. A species' value is determined each instant by the amount of money it has structured relative to the species. Money may be viewed as a fluid that is transferred between competing economic species, its flow pattern being influenced by the value which each economic species attains.

When the economy is in the quiescent steady state, each economic species should be priced at its "correct" value, equilibrium value. Under such conditions, biologic species would maintain their populations at equilibrial levels. However, the steady state among biologic species having predator-prey interactions is the exception rather than the rule, and the same is found to be true in economics. Any slight fluctuation in population (or price) is propagated through time as an oscillation, due to the interactive relationship existing between the species.

797A

The economy may be viewed as an "ensemble" of species whose individual populations (prices) are in perpetual oscillation, much like the oscillation of spatial locations of molecules in a gas. Just as a gas obeys the thermodynamic laws of a canonical ensemble, so too the ecosystem obeys macroscopic dynamic laws. Properties such as temperature, heat flow, heat capacity, and entropy find their analogs in the ecosystem or economy.

Of these properties, the concept most dealt with here will be temperature. Taking, for example, a biologic ecosystem of interacting species, the temperature of a species measures the degree of fluctuation in its population size. The greater the amplitude of these fluctuations, the greater is the specie's temperature. Analogously, the temperature of the stock market is a parameter dependent upon the amplitudes of the market trend fluctuations. (As will be later seen, market temperature may be taken as the macroscopic counterpart to the market factor variance, σ_{rm}.) Depending on the circumstances, temperature may refer either to a particular species in the ensemble or to the ensemble as a whole. Such references may be made interchangeably when the species is in thermodynamic equilibrium with its environment.

Having briefly sketched out the analogy which we are to employ, we will now turn to the field of biology to gain an understanding of the mechanics of predator-prey relationships.

THE VOLTERRA-LOTKA PREDATOR-PREY MODEL

A typical example of a predator-prey relationship observed in nature among hares and lynx is shown in Figure 3, where it is seen that their populations are in oscillation. The basic predator-prey model, proposed independently by both B. Volterra (1931) and A. J. Lotka (1925), assumes for simplicity that the inter-species interaction is restricted to the two species under consideration. Such a situation has been reproduced experimentally in the test tube (see Figure 4-c). The two species under consideration here are two species of paramecium. The predator species, indicated by the dark dotted line, feeds on the prey species, indicated by the white line. The prey species in turn is fed on a constant supply of food. Obviously, if one of the two species becomes extinct, the other will follow suit, and the predator-prey relationship
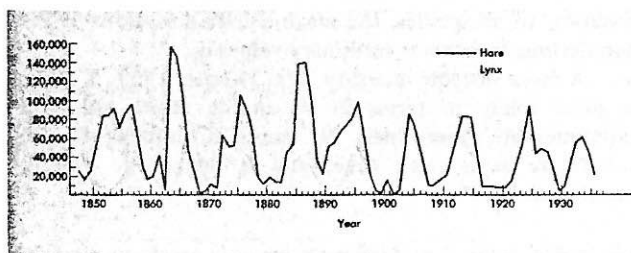
Fig. 3. Oscillations in populations of the snowshoe hare and lynx based on pelts received by the Hudson Bay Company. (From E. J. Kormondy, *Concepts of Ecology*, Prentice-Hall, 1969, p. 96.)
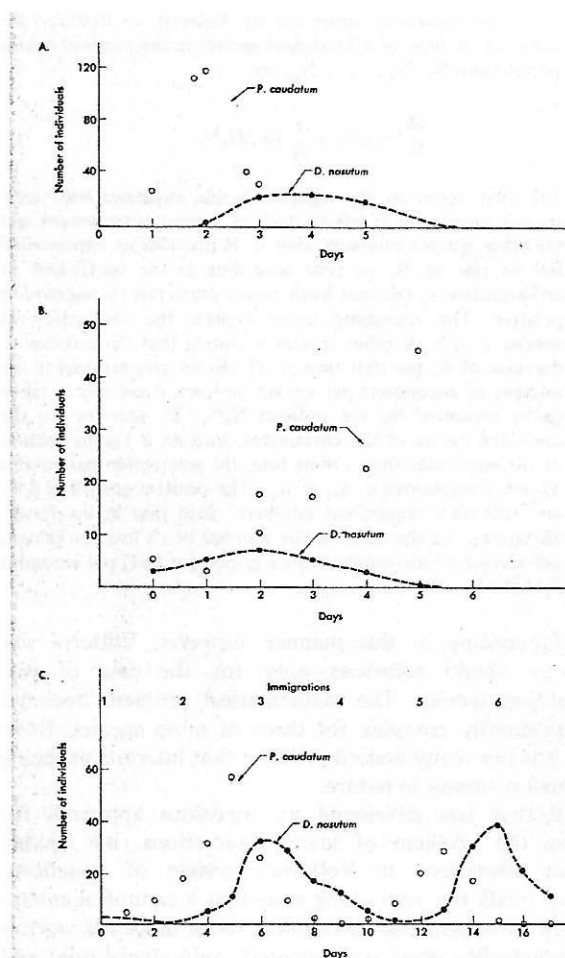
Fig. 4. Prey-predator relationships in two ciliated protozoans. (From E. J. Kormondy, ibid., p. 94.)

will cease to exist. Thus two conditions must be present to permit the existence of a predator-prey relationship: (1) a heterogeneous environment and (2) species migration. Figure 4-a shows the same test tube experiment without a heterogeneous environment and without migration. Figure 4-b shows the same experiment without migration but with a heterogeneous environment. Figure 4-c shows the same experiment performed when both conditions are fulfilled. Immigration was effected by introducing a pair of paramecia into the test tube every three days (indicated by arrows).

One characteristic of the predator-prey relationship is that the population peaks of the prey species preceed those of the predator species. However, in Figure 3, the peaks and troughs of the respective species are in phase. This may be because we are observing the relationship of two species which are embedded in an ecosystem and which are probably interacting with other species.

Edward Kerner (1957, pp. 125-26) describes Volterra's mathematical representation of predator-prey behavior in the following way:

The equations proposed by Volterra to describe the behavior in time of n biological species in interaction, having populations $N_1, N_2, \ldots, N_n$, are

$$\frac{dN_r}{dt} = \epsilon_r N_r + \frac{1}{\beta_r} \sum_s a_{sr} N_s N_r. \qquad (i)$$

The first term on the right-hand side expresses how each species propagates if left to itself in a given environment and no other species interacts with it. It provides an exponential fall or rise of N_r in time according as the coefficient of self-accretion ϵ_r (natural birth minus death rate) is negative or positive. The remaining terms express the interaction of species r with all other species s, stating that the increase or decrease of N_r per unit time is effectively proportional to the number of encounters per second between r and any s, taken to be measured by the product $N_r N_s$. To account for the one-sided nature of the encounters, wherein if r gains because of the encounter then s must lose, the interaction parameters a_{sr} are antisymmetric; $a_{rs} = -a_{sr}$. The positive quantities β_r^{-1} are Volterra's "equivalent numbers" such that in the binary encounters r,s the ratio of the number of s's lost (or gained) per second to the number of r's gained (or lost) per second is $\beta_s^{-1}/\beta_r^{-1}$.

Proceeding in this manner, however, Volterra was able to obtain solutions only for the case of two interacting species. The mathematical problem becomes extraordinarily complex for three or more species. However, it is this many-bodied problem that interests us, being the most common in nature.

Kerner has developed an ingenious approach for solving the problem of many interactions. By making certain alterations in Volterra's system of equations, Kerner treats the interacting system as a canonical ensemble and uses statistical mechanical tools. Since his work is mathematically quite sophisticated, only a very brief and sketchy summary will be presented here. For a more thorough presentation of the mathematics, see Kerner (1957 and 1959).

To begin, a new variable is introduced into Volterra's equations, $v_r = \log N_r/q_r$ where q_r is the steady state value, or average value, of the population size N_r. With this substitution, Eq. (i) is transformed to

$$\beta_r \dot{v}_r = \sum_s a_{sr} q_s (e^{v_s} - 1), \text{ where } \dot{v} = dv/dt. \qquad (ii)$$

When Volterra's equations are represented in this form as a Gibbs microcanonical ensemble, a thermodynamic description can be obtained.

The purpose of having introduced the particular variables v_r is to secure a Liouville's theorem. Consider a large number of copies, a Gibbs ensemble of biological associations each of the same character and each controlled by the same differential equations (ii) but having all variety of initial values of v_r. In the Cartesian space of the v_r (phase space) the configuration of each copy is represented by a point, the ensemble by an ensemble of points. The points are propelled in phase space by the motional equations (ii). When taken to be sufficiently numerous the points constitute a fluid of say, density $\rho (v_1, v_2, \ldots, v_n)$, and velocity $V = (v_1, v_2, \ldots, v_n)$ at this point. Since fluid is neither created nor destroyed we must have the hydrodynamical equation of continuity,

$$\frac{\partial \rho}{\partial t} + \text{div } \rho V \equiv \frac{\partial \rho}{\partial t} + \sum \frac{\partial (\rho \dot{v}_r)}{\partial \dot{v}_r} = 0. \qquad (iii)$$

There follows Liouville's theorem of the conservation of density in phase,

$$\frac{D\rho}{Dt} \equiv \frac{\partial \rho}{\partial t} + \sum \dot{v}_r \frac{\partial \rho}{\partial v_r} = 0 \qquad (iv)$$

stating that as one goes along with the motion of one system point the density in its neighborhood remains invariable. (Kerner 1957, pp. 127-28.)

Eq. (ii) is manipulated to obtain $\sum \beta_r q_r \dot{v}_r (e^{v_r} - 1) = 0$ which upon integration yields a universal single valued constant of motion, G (Kerner 1957, p. 128),

$$G = \sum \tau_r (e^{v_r} - v_r) = \text{constant, where } \tau_r = \beta_r q_r. \qquad (v)$$

We may ask now about the behavior of a part, or component, consisting in, say, only μ of the total n species, of an association. The component does not have its G constant throughout time but exchanges G with the rest of the association, only the total G being conserved. Corresponding to the points on the surface $G = G_0$ in the microcanonical distribution are points in the subspace of dimension μ representing configurations of the component.

How are these component points distributed? The answer is a basic proposition in statistical mechanics: they are distributed according to the law $\rho_\mu = e^{(\psi - G_\mu)/\theta}$, defining Gibbs' canonical ensemble.

The importance of the canonical ensemble in physics comes from the fact that it is a representative ensembel with a capacity for describing not isolated systems with a fixed energy but those which are in thermal equilibrium with their surroundings, continually exchanging energy with them. In the theoretical construction the residual system, that of $n - \mu$ degrees of freedom left over from the original one when the component μ is separated for individual study, holds the position of being the "heat bath" in which the component is immersed. The modulus θ represents the thermodynamic temperature, and ψ the free energy of a system in thermodynamic equilibrium. Through this same door we enter into a 'thermodynamic' description of biological associations. (Kerner 1957, pp. 132-33.)

A derivation for temperature gives (from Kerner 1957, p. 136)

$$\theta = \tau_r/q_r^2 \overline{(N_r - q_r)^2}. \qquad (vi)$$

In other words, the temperature measures, in one number common to all species, the mean square deviations of the populations from their stationary values q_r.

A more testable quantity, $1/x_r$ (Kerner 1959, p. 233), is given solely in terms of N_r and q_r which are both experimental observables, N_r being a population size coordinate at a given time, and q_r being the average population size.*

$$1/x_r = \theta/\tau_r = \overline{(N_r - q_r)^2}/q_r^2 = \overline{(N_r/q_r - 1) \ln (N_r/q_r)}. \qquad (vii)$$

*Note that θ has no subscript since all species are assumed to have the same temperature. This is true of biologic associations in thermodynamic equilibrium, i.e. ones that have been in existence for many cycles. However, x_r is species dependent since it incorporates the factor $\tau_r = \beta_r q_r$ where $x_r = \tau_r/\theta$.

Given that $n_r = N_r/q_r$ (i.e., $\log n_r = v_r$), the probability, $P(n_r) dn_r$, that a species will have its n_r in n_r as opposed to $n_r + dn_r$ is calculated. It is found to be distributed analogously to the Maxwell-Boltzmann distribution which describes the velocity structure of molecules in a gas. For high temperatures, this distribution is skewed; however, at sufficiently small θ it is sensibly normal. (This is opposite to the behavior found in the Maxwell-Boltzmann distribution where the distribution becomes increasingly skewed at lower temperatures.)

Kerner also defines quantities such as the free energy, ψ, and internal energy, $\overline{G}$; quantities such as the "heat capacity", $C = \partial G/\partial \theta$ and the "entropy", $S = \overline{G} - \psi/\theta$. Figures 5 through 8 show plots of these quantities as functions of $1/x_r = \theta/\tau_r$.

These thermodynamic concepts make it easier to understand behavior such as the tendency for biologic associations in non-equilibrium states to decline to equilibrium ones of maximal entropy. Also, the biological analog of heat flow becomes clear.

Zero temperature corresponds to the completely "quiet" stationary state of biological association. The temperature is, so to speak, a kind of indicator of the level of excitation of the association from its stationary state. Its greater significance is, according to an established theorem, that it tells the preferred direction of flow of G from one association to another weakly coupled to it: on the average the association with higher θ will lose G and decrease its θ, and inversely for the low-θ association. (Kerner 1957, p. 136.)

TESTABLE PROPERTIES OF PREDATOR-PREY RELATIONSHIPS

In the previous section we presented an equation for deriving the temperature-like quantity $1/x_r$ from the experimentally observable variables N_r and q_r. Here we will examine a few other observational techniques for deriving x_r, which will serve as a cross check.
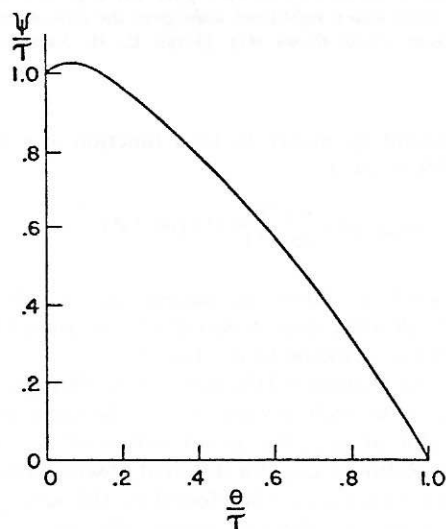
Fig. 5. Free energy per species for fixed τ and variable θ. (From E. H. Kerner, *Bulletin of Mathematical Biophysics*, 19, 1957, pp. 140-43.)
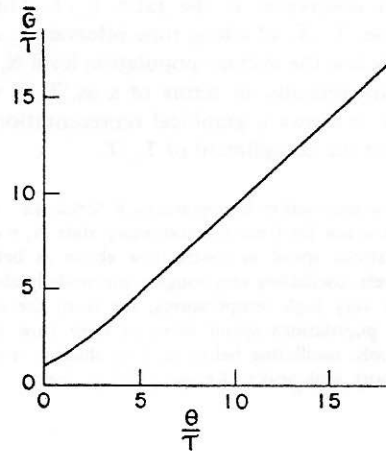
Fig. 6. Single-species contribution to internal energy, giving $\overline{G}$ as function of association temperature θ for fixed intrinsic temperature τ. (From E. H. Kerner, ibid.)
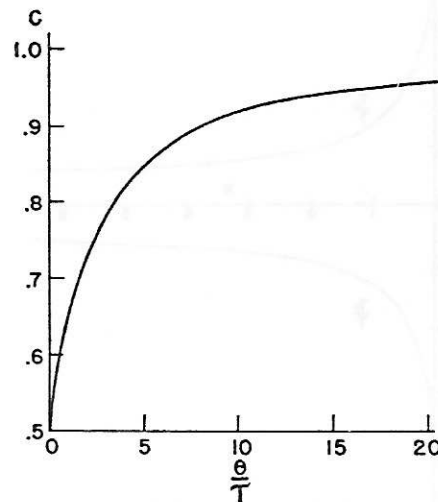
Fig. 7. Heat capacity per species as function of association temperature θ. (From E. H. Kerner, ibid.)
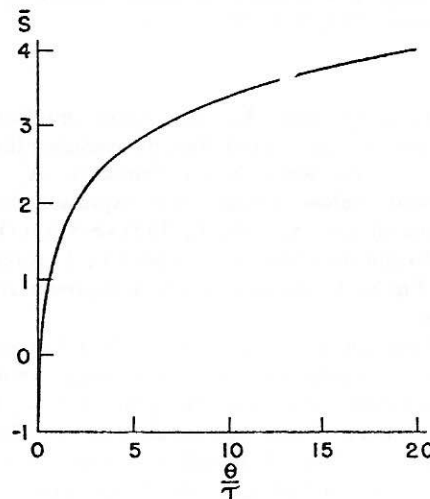
Fig. 8. Entropy per species as function of association temperature θ. (From E. H. Kerner, ibid.)

One such observable is the ratio T_-/T. This is the fraction of time, T_-/T, of a long time interval, T, spent by a population below the average population level $N_r = q_r$. It is expressed theoretically in terms of x as, $T_-/T = I(\sqrt{x}, x - 1)$. (Figure 9 shows a graphical representation.) Note that T_+/T is just the compliment of T_-/T.

At very low association temperatures, $\theta \ll \tau$, i.e., when the association is not far from the stationary state $N_r = q_r$ (all r), the populations spend as much time above as below their average levels, oscillating very roughly sinusoidally about these levels. For very high temperatures, far from the stationary state, the populations spend most of their time at below-average levels, oscillating below in long shallow troughs and above in short, high peaks. (Kerner 1959, p. 238.)

Fig. 10. Mean amplitudes of oscillation above (A_+) and below (A_-) average. (From E. H. Kerner, ibid., p. 240.)

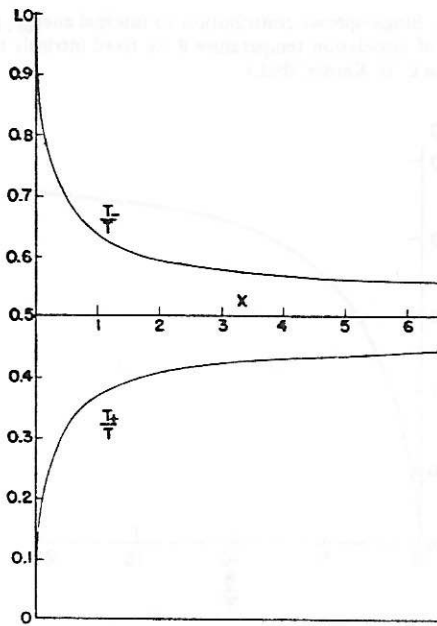Fig. 9. Mean below-average (T_-/T) and above-average (T_+/T) times as functions of x. (From E. H. Kerner, *Bulletin of Mathematical Biophysics*, 1959, 21, p. 238.)
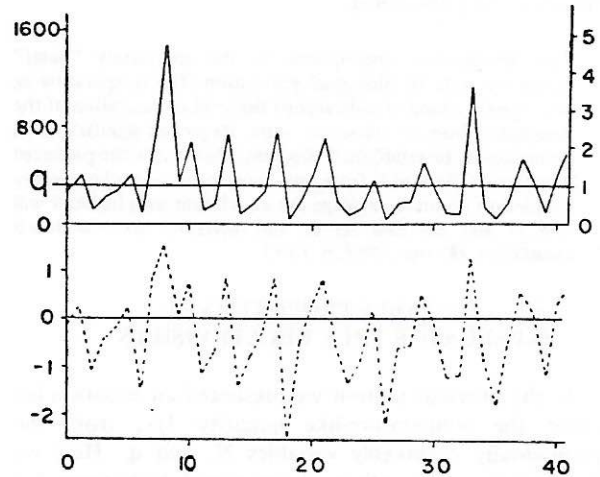
Fig. 11. Labrador fox-catches for the first 40 years of a 91-year period, after Elton (1942). Upper curve gives the catch directly (left-hand scale); right-hand scale gives the reduced variable $n = N/q$. Lower curve shows v(t). (From E. H. Kerner, ibid., p. 249.)

Another observable, A_+, the mean amplitude of oscillation above average, is obtained by averaging the peak differences, $N_r - q_r$, wher. $N > q$. Similarly, A_- is the mean amplitude below average. It is expressed theoretically in terms of x as $A_+ = (1/ T_+/T)(x^x e^{-x}/x!)$ (Kerner 1959, p. 239) and similarly for A_- when (T_+/T) is replaced by (T_-/T). Figure 10 shows a graphical representation of this equation.

By taking an axis of value $\nu = N/q$ horizontally traversing the population curve, one may count the number of times this axis is crossed by the plot (see Figure 11). This frequency of crossings is called $\omega_r(\nu)$. Theoretically, the axis $\nu = N/q = 1$ should be crossed most frequently. We then define $\omega_{rel}(\nu)$ as the ratio of the frequency of crossings about axis $\nu = N/q$, to the frequency of crossings about axis $\nu = 1$. This relative fre-

quency is found by theory to be a function of ν and x_r (Kerner 1959, p. 241),

$$\omega_{rel}(\nu) = \frac{\omega_r(\nu)}{\omega_r(1)} = e^{x_r}(\nu e^{-\nu})^{x_r}. \qquad (viii)$$

The axis $\nu = 1$ is taken as having $\omega_{rel} = 1.00$, and theoretically all other axes chosen should be crossed fewer times, so their ω_{rel} should be less than 1.

From observations of $[(N_r/q_r) - 1] \ln(N_r/q_r)$, T_-/T, A_+, and A_-, a theoretical value, x, may be estimated for x_r, and a plot of $\omega_{rel}(\nu)$ as a function of ν may be constructed. Ratios of ω_{rel} for different observed values of ν (i.e., different axes) may be plotted on this same graph and these may be checked against the theoretically predicted distribution.

For example, in Figure 12 the observed distribution is represented by the connected points and the theoretically
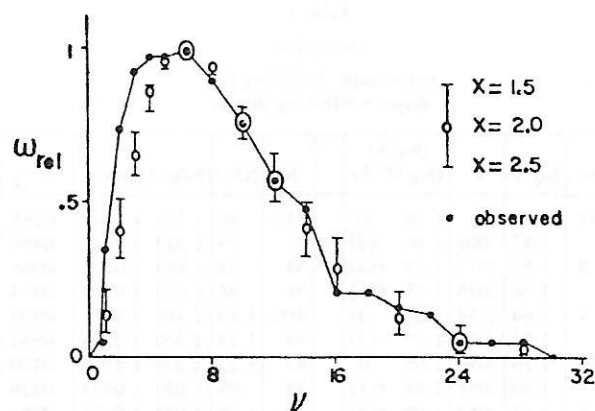
Fig. 12. Comparison of theoretical and observed $\omega_{rel}(\nu)$. Abscissa is ν in units of 0.16, which correspond to intervals of 50 fox-catches in Figure [1] 11. (From E. H. Kerner, ibid., p. 250.)

predicted coordinates appear as vertical limit bars. These limit bars give the (ω, ν) coordinates for a range of x between 1.5 and 2.0, since x is only an estimate calculated from the data.

We have developed here a theoretical statistical tool which may be used to test for the existence of predator-prey relationships in any time series. The next step is to see what kind of results we get when we test stock market fluctuations.

TESTING THE TREND
FLUCTUATIONS OF THE STOCK MARKET

The graph shown in Figure 13 was drawn from a logarithmic plot of the Dow Jones Industrial Average (1897-1952). The average shown prior to the closing of the

market in 1914 consists of 12 industrial stocks and has been adjusted to the postwar average of 20 industrial stocks. The plot shown is a trend line, subjectively drawn through the monthly high-low bars of the original Barron's plot, and is intended to represent a three or four month moving average. (The monthly data points have been omitted for graphical clarity.) Precise correspondence of the trend line with minor market fluctuations is not of vital importance since we are concerned here only with the macroscopic movement of the market average, sample points being sparsely chosen at half-year intervals.

Linear regressions of the trend line were made for the periods (1) 1897-1926, (2) 1924-1938, and (3) 1931-1951, and appear in Figure 13 as three straight lines. Regression coefficients for the first and third periods were both found to be about +0.017 log points per year; for the second period, −0.0069 log points per year. If the third period was chosen to begin in 1942, a much steeper regression coefficient would have been found. We will denote the data points with the symbol log N, and their "average trend values," lying along the regression line, with the symbol log q (t). We may now write the straight line equation for log q (t) as log q = mt + b, where m is the regression coefficient, t is the time coordinate, and b is the y intercept. On a linear graph this would plot as $q = e^{mt + b}$, depicting an exponential long run increase in the market. The period past 1951, not considered in this study, may pose some difficulty for analysis because of the large exponential trend factor.

Given these groups of data points for the market trend fluctuations, we will proceed to calculate the quantities $(N/q - 1) \ln (N/q)$, T_-/T, A_+, and A_- for each period. From these we will estimate an x for each period and draw theoretical plots of $\omega_{rel} (\nu, x)$. Finally, we will check these theoretical plots against the observed ω_{rel}
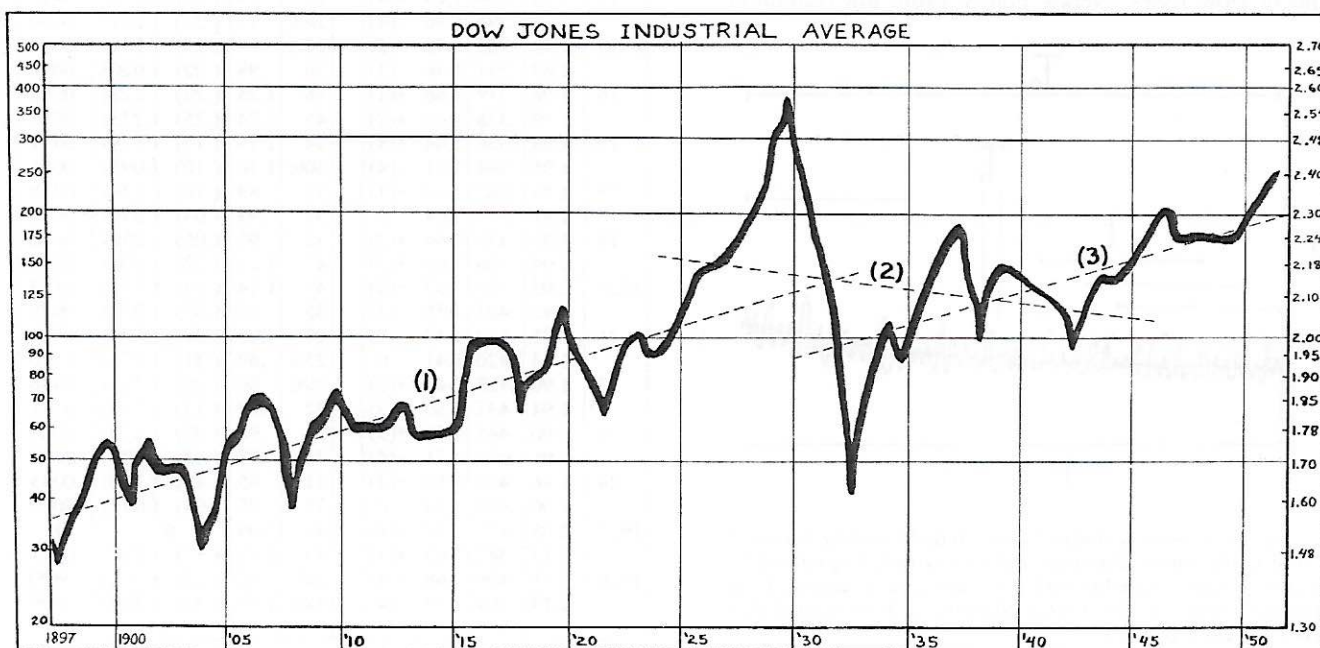

Fig. 13.

PAUL A. LaVIOLETTE

(ν, x) quantities. If there is a good correspondence, we will see this as evidence in support of the hypothesis that market trend fluctuations exhibit predator-prey oscillatory behavior.

The mathematical formulae presented in the last section describe ensembles whose average value q is independent of time. For example, as is seen in Figure 12, q is depicted as a horizontal line independent of t. However, in the present circumstances we have a non-equilibrium ensemble where $G = \Sigma\, \beta_r\, q_r\,(t)\,(e^{v_r} - v_r)$ is a near-constant of motion, i.e., the stock market average, q (t), is exponentially increasing in value (see Figure 14).

To properly calculate x from N_r and q_r (t) we must write Eq. (vii) as,

$$1/x_r = \overline{((N_r/q_r\,(t)) - 1]\, \ln\,(N_r/q_r\,(t))}.\qquad (ix)$$

Each ratio N_r/q_r of the summation is calculated here using a different q_r value. The observables T_-/T, A_+, A_-, and $\omega_{rel}(\nu)$ would then be estimated from analysis of a semi-log plot such as Figure 13.

However, a slightly different approach was taken in this paper: the N_r values have been normalized such that, when plotted on linear paper, their q_r would appear as a horizontal line. This was accomplished by subtracting out the exponential trend factor from the data (see Tables 1, 2, and 3).

The data was read from Figure 13 using a log point scale plotted to the right of the graph. This data was entered in the column marked log N', N' indicating that the data is in an unnormalized condition. From this number the trend factor, δ, is subtracted or added depending on the trend slope (δ is simply the variable, mt, calculated for each value of t.) The normalized data is then entered in the column marked log N, and then translated to N in the adjacent column. It is now in a linear form in terms of Dow Jones Average points. From this column of
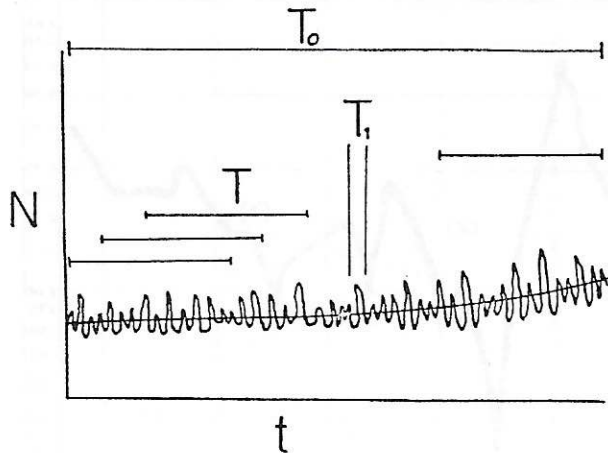

Fig. 14. Schematic diagram illustrating the moving interval T involved in the canonical average, and the relatively long interval T_0 of secular variation, together with the relatively short interval T_1 of population. (From E. H. Kerner, *Bulletin of Mathematical Biophysics*, 23, 1961, p. 152.)

Table I

1897-1926

y intercept: 1.553 log pts.
slope: +.0168 log pts./yr

Date	log N'	δ	(log N) (log N'−δ)		N	N/q	(N/q−1)ℓnN/q		= ⌐
1897	1.50	0	1.50	0	31½	.85	(.15)	(.163)	.0245
	1.47	.008	1.46	+(2)	29	.79	(.21)	(.236)	.0496
8	1.55	.017	1.53	+(3)	34	.92	(.08)	(.083)	.0066
	1.58	.025	1.55	+(5)	36	.98	(.02)	(.020)	.0004
9	1.64	.034	1.61	−(4)	40½	1.10	(.10)	(.095)	.0095
	1.73	.042	1.69	−(2)	49	1.33	(.33)	(.285)	.0941
1900	1.70	.050	1.65	0	45	1.22	(.22)	(.191)	.0420
	1.60	.059	1.54	+(1)	35	.95	(.05)	(.051)	.0026
1	1.67	.067	1.60	+(3)	40	1.08	(.08)	(.077)	.0062
	1.73	.076	1.65	+(4)	45	1.22	(.22)	(.199)	.0438
2	1.65	.084	1.57	−(4)	38	1.03	(.03)	(.030)	.0009
	1.68	.092	1.59	−(2)	39	1.06	(.06)	(.058)	.0035
3	1.68	.101	1.58	−(1)	38	1.03	(.03)	(.030)	.0009
	1.60	.109	1.49	+(1)	31	.84	(.16)	(.174)	.0278
4	1.52	.118	1.40	+(2)	25	.68	(.32)	(.386)	.1235
	1.57	.126	1.44	+(4)	32½	.88	(.12)	(.128)	.0154
1905	1.70	.134	1.57	−(4)	37	1.00	—	—	0
	1.76	.143	1.62	−(3)	41½	1.13	(.13)	(.122)	.0159
6	1.81	.151	1.66	−(1)	45½	1.23	(.23)	(.207)	.0558
	1.84	.160	1.68	0	48	1.30	(.30)	(.262)	.0786
7	1.83	.168	1.66	+(2)	46	1.25	(.25)	(.223)	.0476
	1.75	.176	1.57	+(4)	37½	1.02	(.03)	(.03)	.0009
8	1.63	.185	1.44	+(5)	28	.76	(.24)	(.274)	.0658
	1.73	.193	1.54	−(3)	34½	.94	(.06)	(.062)	.0037
9	1.80	.202	1.60	−(2)	39½	1.07	(.07)	(.067)	.0047
	1.84	.210	1.63	0	42½	1.15	(.15)	(.140)	.0210
1910	1.85	.218	1.63	+(2)	43	1.17	(.17)	(.157)	.0267
	1.78	.227	1.55	+(3)	35½	.96	(.04)	(.041)	.0016
11	1.77	.235	1.53	+(5)	34½	.93	(.07)	(.073)	.0051
	1.77	.244	1.53	−(4)	33½	.91	(.09)	(.094)	.0085
1912	1.76	.252	1.51	−(2)	32	.87	(.13)	(.139)	.0181
	1.82	.260	1.56	0	36½	.99	(.01)	(.010)	.0001
13	1.80	.269	1.53	+(1)	34	.92	(.08)	(.083)	.0066
	1.75	.277	1.47	+(3)	30	.81	(.19)	(.211)	.0401
14	1.75	.286	1.46	+(4)	29	.79	(.21)	(.236)	.0496
	1.75	.294	1.46	−(4)	28½	.77	(.23)	(.261)	.0600
1915	1.76	.302	1.46	−(2)	28½	.77	(.23)	(.261)	.0600
	1.87	.311	1.56	−(1)	36	.98	(.02)	(.020)	.0004
16	1.98	.319	1.66	+(1)	46	1.25	(.25)	(.223)	.0558
	1.99	.328	1.66	+(2)	46	1.24	(.25)	(.223)	.0558
17	1.98	.336	1.64	+(4)	44	1.19	(.19)	(.174)	.0331
	1.95	.344	1.61	−(4)	40½	1.10	(.10)	(.095)	.0095
18	1.85	.361	1.49	−(1)	31	.89	(.16)	(.174)	.0278
	1.91	.370	1.54	0	35	.95	(.05)	(.050)	.0025
19	1.92	.378	1.54	+(2)	35	.95	(.05)	(.050)	.0025
	2.04	.386	1.65	+(4)	45	1.22	(.22)	(.199)	.0438
1920	2.02	.395	1.62	+(5)	42	1.14	(.14)	(.131)	.0183
	1.95	.403	1.55	−(3)	35	.95	(.05)	(.051)	.0026
21	1.88	.412	1.47	−(2)	29½	.80	(.20)	(.223)	.0446
	1.83	.420	1.41	0	25½	.69	(.31)	(.371)	.1150
22	1.90	.428	1.47	+(2)	29½	.80	(.20)	(.223)	.0446
	1.94	.437	1.50	+(3)	32	.87	(.13)	(.139)	.0181
23	2.00	.445	1.55	+(5)	36	.98	(.02)	(.020)	.0004
	1.96	.454	1.51	−(4)	32	.87	(.13)	(.139)	.0181
24	1.96	.462	1.50	−(2)	31½	.85	(.15)	(.163)	.0245
	2.00	.470	1.53	0	35	.95	(.05)	(.051)	.0026
1925	2.05	.479	1.57	+(1)	37	1.00	0		0
	2.12	.487	1.63	+(3)	43	1.17	(.17)	(.157)	.0267
1926	2.18	.496	1.68	+(4)	48½	1.32	(.32)	(.278)	.0890
	2.19	.504	1.69	−(4)	48½	1.32	(.32)	(.278)	.0890

Table II

1924-1938

y intercept: 2.19½ log pts.

slope: −.0069 log pts./yr

Date	log N'	δ	(log N) (log N'+δ)		N	N/q	(N/q−1)	ln N/q	=⌐
1924	1.96	0	1.96	0	91	.54	(.46)	(.777)	.358
	2.00	.003	2.00	+(3)	101	.60	(.40)	(.916)	.368
1925	2.05	.007	2.06	−(3)	114	.68	(.32)	(.386)	.124
	2.12	.010	2.13	0	135	.80	(.20)	(.223)	.045
6	2.18	.014	2.19	+(4)	156	.93	(.07)	(.073)	.005
	2.19	.017	2.21	−(3)	161	.96	(.04)	(.041)	.002
7	2.20	.021	2.22	+(1)	166	.99	(.01)	(.010)	0
	2.24	.024	2.26	+(4)	184	1.10	(.10)	(.095)	.009
8	2.30	.028	2.33	−(2)	213	1.27	(.27)	(.239)	.065
	2.36	.031	2.39	+(1)	246	1.46	(.46)	(.378)	.174
9	2.45	.035	2.49	−(5)	305	1.81	(.81)	(.593)	.480
	2.53	.038	2.57	−(2)	370	2.20	(1.20)	(.788)	.946
1930	2.48	.041	2.52	+(1)	332	1.97	(.97)	(.678)	.657
	2.37	.045	2.42	−(5)	260	1.55	(.55)	(.438)	.241
1	2.25	.048	2.30	−(2)	199	1.18	(.18)	(.166)	.030
	2.14	.052	2.19	+(2)	156	.93	(.07)	(.073)	.005
2	1.95	.055	2.01	−(5)	101	.60	(.40)	(.916)	.366
	1.65	.059	1.71	−(1)	51	.30	(.70)	(1.20)	.840
3	1.82	.062	1.88	+(2)	76	.45	(.55)	(.799)	.440
	1.92	.066	1.99	−(4)	97	.58	(.42)	(.545)	.229
4	2.02	.069	2.09	−(1)	123	.73	(.27)	(.315)	.081
	1.98	.072	2.05	+(2)	113	.67	(.33)	(.400)	.131
1935	2.01	.076	2.09	−(4)	122	.73	(.27)	(.315)	.085
	2.07	.079	2.15	−(1)	141	.84	(.16)	(.174)	.028
6	2.15	.083	2.23	+(3)	171	1.02	(.02)	(.020)	.004
	2.20	.086	2.29	−(4)	193	1.15	(.15)	(.140)	.021
7	2.25	.090	2.34	0	218	1.30	(.30)	(.262)	.079
	2.24	.093	2.33	+(3)	215	1.28	(.28)	(.247)	.069
1938	2.10	.097	2.20	−(3)	157	.93	(.07)	(.073)	.005
	2.07	.100	2.17	0	148	.88	(.12)	(.128)	.053

Table III

1931-1951

y intercept: 1.965 log pts.

slope: +.0167 log pts./yr

Date	log N'	δ	(log N) (log N'−δ)		N	N/q	(N/q−1)	ln N/q	=⌐
1931	2.25	0	2.25	0	178	1.84	(.84)	(.610)	.5124
	2.14	.008	2.13	+(2)	135	1.39	(.39)	(.329)	.1283
32	1.95	.017	1.93	+(3)	86	.89	(.11)	(.117)	.0129
	1.65	.025	1.62	+(5)	47	.48	(.52)	(.734)	.3817
33	1.82	.034	1.79	−(4)	61	.63	(.37)	(.462)	.1709
	1.92	.042	1.88	−(2)	75	.77	(.23)	(.261)	.0600
34	2.02	.050	1.97	0	93	.96	(.04)	(.041)	.0016
	1.98	.059	1.92	+(1)	83	.86	(.14)	(.150)	.0210
1935	2.01	.067	1.94	+(3)	88	.91	(.09)	(.094)	.0085
	2.07	.076	1.99	+(4)	99	1.02	(.02)	(.020)	.0004
36	2.15	.084	2.07	−(4)	117	1.21	(.21)	(.182)	.0382
	2.20	.092	2.11	−(2)	128	1.32	(.32)	(.278)	.0890
37	2.25	.101	2.15	−(1)	141	1.45	(.45)	(.372)	.1674
	2.24	.109	2.13	+(1)	135	1.39	(.39)	(.329)	.1283
38	2.10	.118	1.98	+(3)	96	.99	(.01)	(.010)	.0001
	2.07	.126	1.94	+(4)	88	.91	(.09)	(.094)	.0085
39	2.15	.134	2.02	−(4)	104	1.07	(.07)	(.068)	.0048
	2.16	.143	2.02	−(3)	104	1.07	(.07)	(.068)	.0048
1940	2.16	.151	2.01	−(1)	103	1.06	(.06)	(.058)	.0035
	2.14	.160	1.98	0	96	.99	(.01)	(.010)	.0001
41	2.11	.168	1.94	+(2)	88	.91	(.09)	(.094)	.0085
	2.09	.176	1.91	+(4)	82	.85	(.15)	(.163)	.0245
42	2.05	.185	1.86	+(5)	73	.75	(.25)	(.288)	.0720
	2.01	.193	1.82	−(3)	66	.68	(.32)	(.386)	.1235
43	2.08	.201	1.88	−(1)	76	.78	(.22)	(.248)	.0546
	2.14	.210	1.93	0	85	.88	(.12)	(.128)	.0154
44	2.14	.218	1.92	+(2)	84	.87	(.13)	(.139)	.0181
	2.15	.227	1.92	+(3)	84	.87	(.13)	(.139)	.0181
1945	2.18	.235	1.94	+(5)	88	.91	(.09)	(.094)	.0085
	2.22	.244	1.98	−(4)	95	.98	(.02)	(.020)	.0004
1946	2.28	.252	2.03	−(2)	107	1.10	(.10)	(.095)	.0095
	2.31	.260	2.05	0	112	1.15	(.15)	(.140)	.0210
47	2.25	.269	1.98	+(1)	96	.99	(.01)	(.010)	.0001
	2.25	.277	1.97	+(3)	94	.97	(.03)	(.030)	.0009
48	2.25	.286	1.96	+(4)	92	.95	(.05)	(.05)	.0025
	2.25	.294	1.96	−(4)	90	.93	(.07)	(.073)	.0051
49	2.25	.302	1.95	−(2)	89	.92	(.08)	(.083)	.0066
	2.24	.311	1.93	−(1)	85	.88	(.12)	(.128)	.0154
1950	2.30	.319	1.98	+(1)	96	.99	(.01)	(.010)	.0001
	2.33	.328	2.00	+(2)	100	1.03	(.03)	(.030)	.0009
1951	2.37	.336	2.03	+(4)	108	1.11	(.11)	(.104)	.0114
	2.40	.344	2.06	−(4)	114	1.18	(.18)	(.166)	.0299

N, a mean is determined and called q ($q = \bar{N}$). Next, the ratio N/q is calculated for each data point N, q remaining invariant. Finally, the quantities $(N/q - 1)\ln(N/q)$ are determined for each value of N/q. The mean determined from this column is the quantity $1/x$. Whereupon, the inverse temperature-like quantity x is determined to be 34, 5, and 19 in each of the three periods studied. Figures 15-a,b,c show a linear plot of the normalized data N vs. time. The horizontal line in each case indicates q.

The advantage of the normalization approach is that the data can be displayed on a linear plot and the quantities T_-/T, A_+, A_-, and ω_{rel} (ν) can be more easily read from the graph. However, it must be realized that this could potentially introduce a certain amount of error into the data. So, to put this approximation in perspective, the values of x were calculated for the first two periods using formula (ix), the "proper method", and found to be 30½ and 5.1½. When these are compared with the values of 34½ and 5.0½, determined by the normalization approach, we see that the approximation is within reason.

The calculations of A_+, A_-, and T_-/T for the three periods are shown in Tables 4, 5, and 6 under subheadings (1) and (2). These values were translated into predictions of x using Figures 9 and 10, and were entered under subheadings (3) along with the N/q determinations.

Next, theoretical and observed (ω_{rel}, ν) coordinates were determined and tabulated under subheadings (4). The estimated value of x, determined from subheadings (3), was used in formula (viii) to determine the theoretical (ω_{rel}, ν) coordinates for each period. For example, for the first and third periods, x was estimated to be around 20. So, then taking the log of formula (viii), we get log (ω_{rel}) = 20 + 20 (log $\nu - \nu$). Substituting a range of ν into the equation, a theoretical distribution of ω_{rel} is determined.

The observed relative frequency data was obtained in the following way. Various horizontal ν axes were chosen
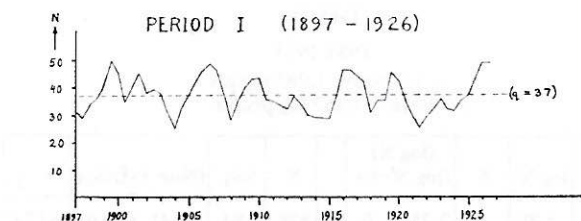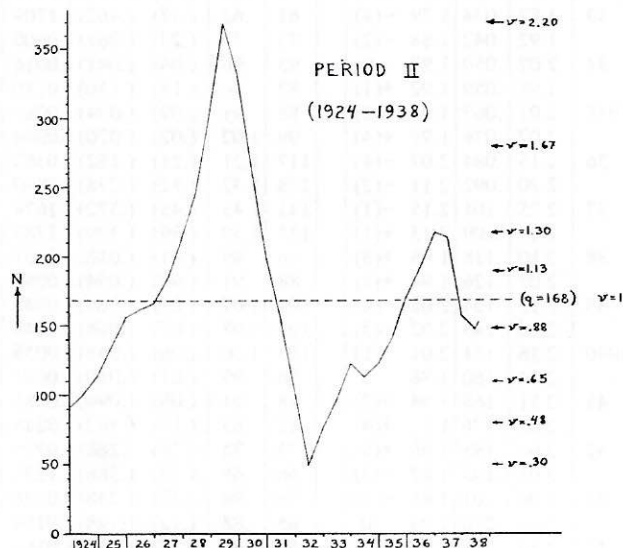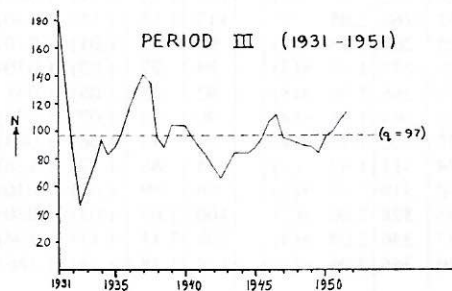
Fig. 15-a.

Fig. 15-b.

Fig. 15-c.

Table IV

Calculations—Period I (1897-1926)

(1) $A_+ = \overline{N - q}/q$		(q = 37)		$A_- = \overline{q - N}/q$	
N	N - q			N	q - N
49	12			29	8
45	8	$A_+ = 9/37 = .24$		35	2
48	11			25	12
43	6	$A_- = 7.56/37 = .20$		27	10
46	9			32	5
45	8			28½	8½
	6|54			31	6
	9			25½	11½
				31½	5½
					9|68
					7.56

(2) $T_-/T = 33/60 = .55$

(3): Estimation of x

(a) $\left.\begin{array}{c} A_+ \\ A_- \end{array}\right\} \longrightarrow x > 15$

(b) $T_-/T \longrightarrow x > 15$

(c) $\left(\dfrac{N}{q} - 1\right) \ell n \dfrac{N}{q} \longrightarrow x = 34$

(4): (ω_{rel}, ν) coordinates

theoretical ω_{rel}			observed ω_{rel}	
(for x = 20)				
ν	ω_{rel}		ν	ω_{rel}
.5	.05½		.67	.08
.6	.19		.76	.39
.7	.42		.79	.69
.8	.70½		.81	.77
.9	.92		.84	.85
1.0	1.00		.94	1.15
1.1	.92		1.00	1.00
1.2	.76½		1.10	1.00
1.3	.56½		1.16	.92
1.4	.38		1.22	.77
1.5	.24		1.24	.46
			1.30	.31
			1.34	.08

to traverse the plots shown in Figures 15-a,b,c. For example, see Figure 15-b where the axes have been indicated by dashes to the right of the plot. The number of times that a particular axis ($\nu = N/q$) was traversed by the plot was recorded as the ω_r for that axis. A tangency was considered as a single crossing. The ratio $\dfrac{\omega_r(\nu)}{\omega_r(1)}$ was then calculated for each axis to determine its ω_{rel} value.

Figures 16-a,b,c show a comparison of theory with observation for the (ω_{rel}, ν) coordinates. Period 1 shows a close approximation between theory and observation; period 2, a very close approximation; period 3, not as close. Two theoretical curves were plotted for period 2 to show how the shape of the theoretical ω_{rel} distribution

changes as a function of x. Period 3, a transition period for the stock market, spans both the high temperature period of the 30's and the low temperature period of the 40's, so the odd shape of plot-b does not come as a surprise. Period 1 (see Figure 15-a) is a good example of an ensemble in thermodynamic equilibrium. Note the gaussian shape to the plot observed in Figure 16-a. Theory predicts this distribution to be sensibly normal at low temperatures such as x = 20. Period 2, the period of the great stock market crash, shows an unusually close correspondence to theory despite the relatively few cycles available for data analysis. The plot in Figure 16-b shows a skewed ω_{rel} plot characteristic of a high temperature distribution.

In conclusion, the results here show strong evidence of predator-prey behavior in stock market trend fluctuations. Also, a high temperature period is evident between 1924 and 1938, during which time the market appears to have temporarily deviated by a significant amount from

Table V

Calculations—Period II (1924-1938)

(1) $A_+ = \overline{N - q/q}$ (q = 168) $A_- = \overline{q - N/q}$

N	N - q		N	q - N
370	202	$A_+ = 126/168 = .75$	51	117
218	50	$A_- = 86/168 = .51$	113	55
	2⌐252			2⌐172
	126			86

(2) $T_-/T = 8\frac{1}{4}/15 = .55$

(3) Estimation of x

(a) $\left.\begin{array}{c} A_+ \\ A_- \end{array}\right\} \longrightarrow x = 1.7$

(b) $T_-/T \longrightarrow x > 15$

(c) $\left(\dfrac{N}{q} - 1\right) \ln \dfrac{N}{q} \longrightarrow x = 5$

(4): (ω_{rel}, ν) coordinates

ν	ω_{rel} theoretical (x = 2)	(x = 5)	ω_{rel} observed
.30	.36½	.08	.25
.48	.65	.34	.50
.65	.85	.67	.75
.88	.98½	.96	1.00
1.00	1.00	1.00	1.00
1.13	.98½	.96	1.00
1.30	.92½	.83	.75
1.50	.82½	.62	—
1.67	.73	.45½	.50
2.00	.54	.15½	—
2.20	.36	.08	.25

Table VI

Calculations—Period III (1931-1951)

(1) $A_+ = \overline{N - q/q}$ (q = 97) $A_- = \overline{q - N/q}$

N	N - q		N	q - N
141	44	$A_+ = 22/97 = .23$	47	50
104	7		83	14
112	15		88	9
	3⌐66	$A_- = 23/97 = .24$	66	31
	22		85	12
				5⌐116
				23

(2) $T_-/T = 12\frac{1}{2}/21 = .59$

(3): Estimation of x

(a) $\left.\begin{array}{c} A_+ \\ A_- \end{array}\right\} \longrightarrow x > 15$

(b) $T_-/T \longrightarrow x = 2\frac{1}{2}$

(c) $\left(\dfrac{N}{q} - 1\right) \ln \dfrac{N}{q} \longrightarrow x = 19$

(4): (ω_{rel}, ν) coordinates

(theoretical ω_{rel} for x = 20 ⟶ see table IV)

observed ν	ω_{rel}
.49	.13
.62	.25
.68	.38
.76	.50
.85	.62
.88	.87
.91	1.12
1.00	1.00
1.07	.87
1.11	.75
1.15	.62
1.24	.38
1.44	.25

the near equilibrium regime. This suggests that the market crash in 1929 may not have been produced by exogenous factors but was caused by a temporary failure of the stock market to properly price itself relative to the economy. Thus there is reason to believe that, during the period 1926-1929, the stock market as an aggregate has shown a period of excessive inefficiency.

Efficient market theory would maintain that, during this period, individual stocks were efficiently priced relative to the whole market. However, when our frame of reference shifts from the micro level to the macro level, a different picture emerges. The stock market as a whole during this period appears to have behaved as a "hot issues market" where the preceeding inflated market prices appeared to foster further market inflation, driving the market into a temporary inflationary spiral.
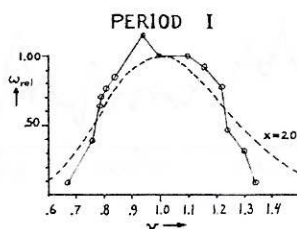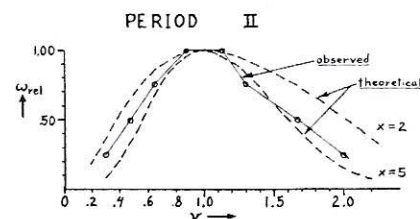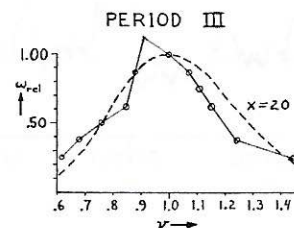

Fig. 16-b.

Fig. 16-a.

Fig. 16-c.

TEMPERATURE AND MARKET VARIABILITY

The concept in present financial theory which most closely corresponds to the market temperature θ is the market factor variance σ_{rm}^2.

$$\sigma_{rm}^2 = \frac{\Sigma \, (R_t - \bar{R}_t)^2}{n} \qquad \text{where } R_t = P_t/P_{t-1} \qquad \text{(x)}$$

$$P \equiv \text{value of the market.}$$

This is compared to $1/x_r = \theta/\tau_r$, which is

$$\theta/\tau_r = \frac{\Sigma \, (N_r - q_r)^2}{nq_r^2} \qquad \text{where } q_r = \bar{N}_r.$$

The P_{t-1} factor in the denominator of every R_t term in (x) tends to serve the same purpose as the q_r^2 term in (xi). Both normalize the summation with respect to the ambient level of the market. Although this is done statistically in different ways, we may still write, without loss of rigor, that $1/x_r$ is proportional to $(\sigma_{rm})^2$.

Let us verify this. Figure 17 shows the one year σ_{rm} plotted from 1897 to 1969. Figure 18 shows the ten year plot of σ_{rm} during the depression. From these we estimate σ_{rm} for the first period (1897-1926) to be about 0.04½; for the second period (1924-1938), about 0.12. Taking the ratio of the squares, we get $(\sigma_1/\sigma_2)^2 = (.12/.04\tfrac{1}{2})^2 = 7.2$. Taking the ratio of the $1/x$ quantities for those same periods, we get $(1/x_1)/(1/x_2) =$ either $.2/.029 = 6.9$, or
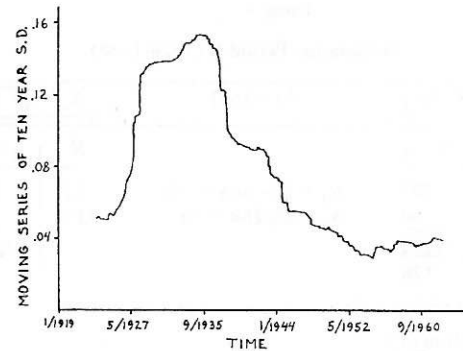
Fig. 18.

$.2/.0327 = 6.1$, depending on whether Eq. (vii) or (ix) was used for the calculation. Thus there appears to be a fairly close correspondence between $1/x_r$ and σ_{rm}^2.

Robert Officer, in his (University of Chicago) doctoral dissertation, part 2, entitled "The Variability of the Market Factor of the New York Stock Exchange," deals with the question why there has been a decline in the market factor variability since 1926. He presents arguments against a number of traditional explanations, these being (1) the formation of the SEC, (2) the establishment of margin requirements, (3) the changing composition of stocks (types of companies or enterprises) listed on the N.Y.S.E.

He studied the relationship between industrial production and the market factor, using autocorrelation
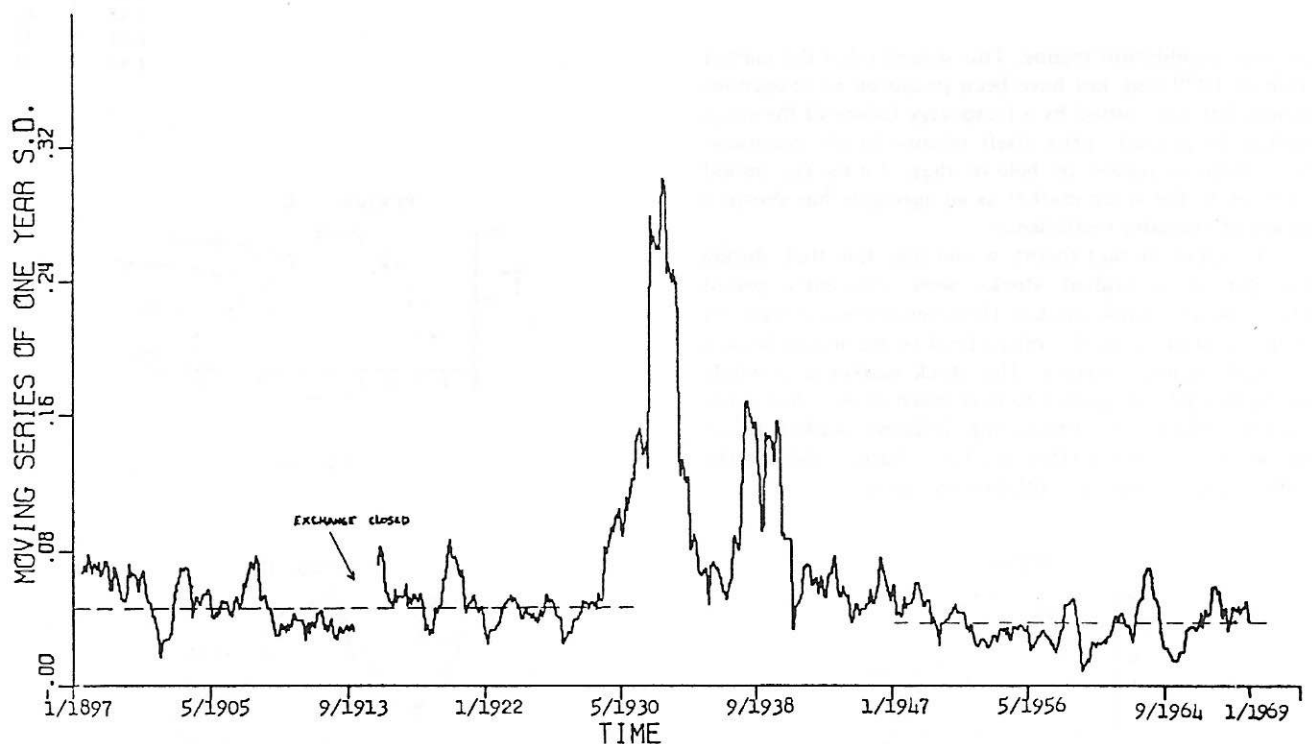

Fig. 17. The behavior of the one year standard deviation of the monthly returns of the market factor. (From R. R. Officer, University of Chicago Graduate School of Business, dissertation, 1971.)

models, and found that the time series behavior of industrial production relatives during the period 1929-1944 was distinctly different from that of the periods 1919-1929 and 1944-1969; moreover, that the autocorrelation models for these latter two periods were identical with almost identical values for the parameters. He states (Officer 1971, p. 28):

> This evidence supports the hypothesis that the return to normality of the market factor after the 1930's reflected the economy as a whole and not any action taken with the specific aim of regulating the behavior of the market.

Earlier in his paper he notes (p. 10):

> ...the variability of the market factor before the 1930's is similar to that after about 1942. It is apparent that the decline in variability observed by other studies* is better described as a return to normal levels of variability after a period of abnormal behavior in the 1930's.

BIO-ECONOMIC IMPRESSIONS

As a sequel to Officer's observations, the following hypothesis is presented, based on our bio-economic analogy of stock market behavior. It is here believed that the market and economy were in thermodynamic equilibrium at a low prevailing temperature prior to period 2 (1924-1938); that during this period a significant and rather sudden change occurred in the stock market, whereby its temperature increased by 6 or 7 fold. This high level of excitation, being exhibited by the extreme over-valued and under-valued swings of the market, indicates that the "internal energy," G, of the market had been raised. The subsequent, rather rapid, decay of these fluctuations to their initial temperature at thermodynamic equilibrium indicates that during the high temperature period the market was in a non-equilibrium, inefficient state with respect to the rest of the economy. Due to the coupling of the stock market with other species in the economy, a phenomenon of "heat flow" took place, whereby the internal energy of the stock market became dissipated into the surrounding economy. As a result, the whole system proceeded towards a state of maximal entropy, thermodynamic equilibrium. Thus, as the stock market temperature dropped, the temperature of its associated species (i.e. bond markets, banks, accounts payable, currency exchanges, foreign money markets) was temporarily increased. Figure 19 shows how other economic species were affected by the crash. Note the strong coupling with the corporate bond market.

As the world economy gradually recovered from the effects of the depression, thermodynamic equilibrium was
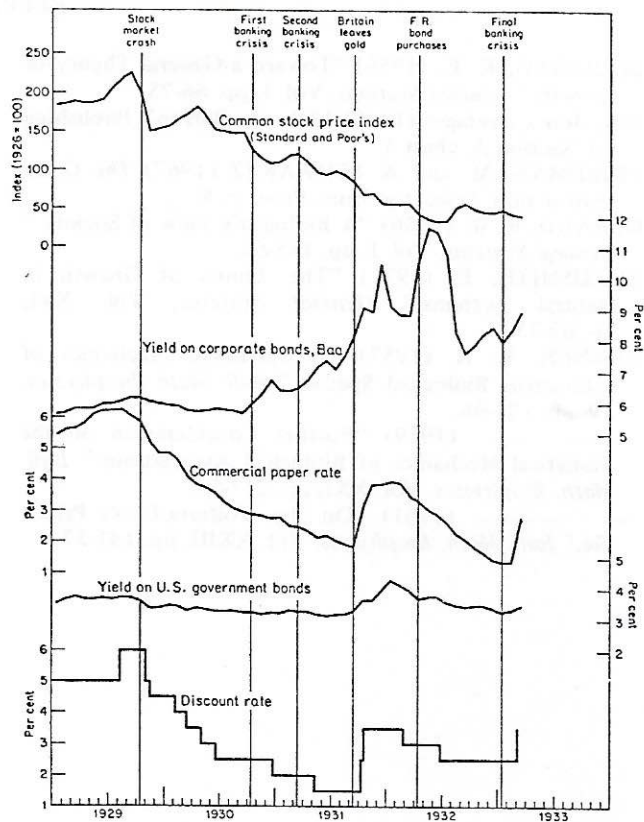
SOURCE: Common stock price index, Standard and Poor's, as published in *Common-Stock Indexes, 1871-1937* (Cowles Commission for Research in Economics, Bloomington, Ind., Principia Press, 1938), p. 67. Discount rates, *Banking and Monetary Statistics*, p. 441. Other data, same as for Chart 35.

Fig. 19. Comn.on stock prices, interest yields, and discount rates of Federal Reserve Bank of New York, monthly, 1929-March 1933. (From M. Friedman and A. Schwartz, *The Great Contraction*, Princeton University Press, 1967, p. 8.)

once again restored. However, the depression had, as a result, a considerable erroding effect on the carrying capacity of the economy. Note the downward offset in the market regression line in Figure 13.

One question which arises is, why did the market become so over-valued during the period 1926-1929? Why did it continue to be bullish when, according to the prevailing market temperature at that time, a bearish market was in order? In other words, why did the New York Stock Exchange become a hot market? In seeking an explanation of the great crash, historians and economists may do well to focus on this pre-crash period. They may find this to be a classic example where the functioning of our economic system spontaneously deviated from its expected behavior as a result of the unpredictable human element.

*King, "Market and Industry Factors in Stock Price Behavior," 139-190; Blume, "The Assessment of Portfolio Performance," 1968; Fisher and Lorie, "Some Studies of Variability of Returns," 99-134.

REFERENCES

BOULDING, K. E. (1956) "Toward a General Theory of Growth." *General Systems*, Vol. I, pp. 66-75.

Dow Jones Averages (1948) 13th ed., Barron's Publishing Co. Section A, chart 3.

FRIEDMAN, M. and A. SCHWARTZ (1967) *The Great Contraction,* Princeton Univ. Press, p. 8.

GERARD, R. W. (1956) "A Biologist's View of Society." *General Systems,* Vol. I, pp. 155-62.

GOLDSMITH, E. (1971) "The Limits of Growth in Natural Systems." *General Systems*, Vol. XVI, pp. 69-73.

KERNER, E. H. (1957) "A Statistical Mechanics of Interacting Biological Species." *Bull. Math. Biophysics,* 19, pp. 121-96.

——————. (1959) "Further Consideration on the Statistical Mechanics of Biological Associations." *Bull. Math. Biophysics*, Vol. XXI, pp. 217-55.

——————. (1961) "On the Volterra-Lotka Principle." *Bull. Math. Biophysics,* Vol. XXIII, pp. 141-57.

KORMONDY, E. J. (1969) *Concepts of Ecology.* Englewood Cliffs, N.J.: Prentice-Hall.

LORIE, J. and M. HAMILTON. (1973) *The Stock Market, Theories and Evidence.* Homewood, Ill.: Richard D. Erwin.

OFFICER, R. R. (1971) "An Examination of the Time Series Behavior of the Market Factor of the New York Stock Exchange." Paper No. 2 in "The Variability of the Market Factor of the New York Stock Exchange." Unpublished dissertation, University of Chicago Graduate School of Business.

SHISKIN, J. (1968) "Systematic Aspects of Stock Price Fluctuations." In *Modern Developments in Investment Management,* J. Lorie and R. Brealey, eds. New York: Praeger, pp. 670-88.

SPENCER, H. (1906) *Principles of Sociology* Part II, Ch. II: "A Society is an Organism," section 215-216.